Einstein on mass and energy

Eugene Hecht[a)]
Department of Physics, Adelphi University, Garden City, New York 11530

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This paper explores the evolution of Einstein's understanding of mass and energy. Early on, Einstein embraced the idea of a speed-dependent mass but changed his mind in 1906 and thereafter carefully avoided that notion entirely. He shunned, and explicitly rejected, what later came to be known as "relativistic mass." Nonetheless many textbooks and articles credit him with the relation $E=mc^2$, where E is the total energy, m is the relativistic mass, and c is the vacuum speed of light. Einstein never derived this relation, at least not with that understanding of the meaning of its terms. He consistently related the "rest energy" of a system to its invariant inertial mass. © *2009 American Association of Physics Teachers.*
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I. INTRODUCTION

Einstein's first paper on relativity[1] appeared when the concept of a speed-dependent electromagnetic mass had already become a topic of considerable interest. He accepted this idea but changed his mind after being confronted by a far more compelling insight. We will show that after reading Planck's 1906 article[2] in which the concept of relativistic momentum was introduced, Einstein came to realize that it was the relativistic equations for energy and momentum that were primary. From that perspective it became clear that the inertial mass m was invariant, and he never again spoke of mass as being speed dependent.

Over the next several years, no doubt unaware of Einstein's change of mind, a number of researchers continued to elaborate on the idea that inertial mass varied with relative speed v. For them Newtonian mass had to be replaced by the idea of "relativistic mass" $m_r(v)$, where

$$m_r = m_0(1 - v^2/c^2)^{-1/2}. \tag{1}$$

Here m_0 is the object's *rest mass*, the inertial mass when $v = 0$. It was already commonplace to represent $(1-v^2/c^2)^{-1/2}$ by γ, whereupon $m_r = m_0\gamma$, or just $m_r = m\gamma$. After 1908 there were two conflicting interpretations of relativistic dynamics: Einstein's invariant-mass perspective and the relativistic mass formulation.[3]

Meanwhile Einstein had shown that the energy of a system at rest was proportional to its inertial mass. Over the decades that followed, this extremely significant discovery took on the symbolic form $E=mc^2$, wherein E is the total energy and m is the relativistic mass. Surprisingly, Einstein never derived nor ever accepted this relation. As $E=mc^2$ was becoming the most widely recognized symbol of the Atomic Age, Einstein maintained that this general statement was formulated "somewhat inexactly." [4]

II. ELECTROMAGNETIC MASS

In 1881, J. J. Thomson, the discoverer of the electron, raised the possibility that inertia might, in whole or in part, be electromagnetic in origin.[5] The implications of his analysis captured the imagination of many researchers and led to the publication of over a hundred related articles.[6] In 1897 George F. C. Searle derived expressions for the electromagnetic energy possessed by various moving charge distributions.[7] Although he did not explicitly calculate it,[8] his treatment ultimately led to an equation for a speed-dependent electromagnetic mass.[9]

The next development came in 1899 in an article[10] by the renowned physicist Hendrik Antoon Lorentz. That paper dealt with a stationary aether, vibrating media, and moving "small charged particles or ions." Lorentz showed "that the same ion will have different (speed-dependent) masses for vibrations parallel and perpendicular to the velocity of translation."

The first experimentalist to take up the challenge of measuring the predicted variation in the mass of moving charged objects was Walter Kaufmann. He began his study in 1901 using high-speed electrons emitted from radium.[9] Kaufmann accelerated the electrons laterally with a localized electric field **E**, bending their paths into circular arcs using a constant magnetic field **B** parallel to **E**. By measuring the point of impact on a film plate and knowing the values of both fields, he calculated the charge-to-mass ratio (e/m) of the electron and found that it appeared to be speed dependent.[11] The geometry of Kaufmann's apparatus was a little complicated, but the basic physics was straightforward:[12] the B-field bent the beam into arcs whose radii depended on each electron's momentum. The known E-field allowed the particles' speeds to be determined.

Although no one knew it at the time, the definition of the momentum p was crucial. If the traditional expression $p = mv$ was used, and there was no reason not to, the experiment clearly showed that e/m decreased as the speed increased and the mass was speed dependent. If the proper relativistic momentum $p=\gamma mv$ was used (which would have been impossible because it had not yet been conceived), the experiment would have shown that e/m was constant as was m; it was p that was changing with v in a nonlinear way.

Kaufmann utilized Searle's analysis and determined that "the formula gives the observed values (of the speed-dependent mass) quite well." [9] In later papers he would correct several early errors and use a more sophisticated analysis than Searle's.

The theoretician Max Abraham, who was a friend of Kaufmann, modeled the electron as a uniformly charged rigid sphere moving in the all-pervasive aether. He called the mechanical mass m and the electromagnetic mass μ. That notation would show up again in Einstein's June 1905 paper.[13] Abraham[14] conceived of both a transverse and a longitudinal mass distinguished by the direction of the electron's velocity with respect to its acceleration. He expressed these two speed-dependent masses in terms of μ_0, the electromagnetic mass for small velocities.

Lorentz in 1904 modeled the electron as a tiny charged sphere,[15] which experienced a FitzGerald contraction as it moved through the aether. He was aware of Abraham's 1903 analysis and derived a transverse mass that was equivalent to $m_0\gamma$—which is identical to Eq. (1)—and a longitudinal mass corresponding to $m_0\gamma^3$. Both of his results were much simpler than Abraham's.[14]

Kaufmann, now in the middle of a significant theoretical dispute, continued his research hoping to resolve the matter. In his setup each electron moved at a constant speed in an almost circular arc perpendicular to **B**. Each experienced an acceleration that was perpendicular to its velocity, and therefore the quantity accessible to Kaufmann was the transverse mass. As time went on, Lorentz's model came to fit Kaufmann's observations almost, but not quite, as well as did Abraham's.[16]

Kaufmann and those who improved upon and extended his work were convinced that they had observed the manifestations of mass varying with speed. Einstein would later allude to the alternative that they had instead observed the manifestations of momentum varying nonlinearly with speed.

III. THE SPECIAL THEORY

Einstein was an assistant examiner at the Federal Office for Intellectual Property, the patent office, in Bern, Switzerland, when he published his first article on relativity (June 1905). "Zur Elektrodynamik bewegter Körper" was no routine analysis applying Maxwell's theory to yet another set of specific conditions.[17] Rather it was an entirely new way to look at the physical world. Not many initially recognized just how revolutionary it was.[18] The final portion of the paper was the "Electrodynamic Part," the last section of which, the "Dynamics of the Slowly Accelerated Electron," is the treatment we are interested in here.

Young Einstein (he had just turned 26) took a remarkably general approach to the moving electron. He applied his new theory without making any assumptions about the structure of the electron. By using $\mathbf{F}=m\mathbf{a}$ he showed (in about 2.5 pages) that a pair of velocity-dependent transverse and longitudinal mass equations could be derived, as he put it, "following the usual approach."

His conclusions were presented in a way equivalent to *transverse mass* $=\mu(1-v^2/c^2)^{-1}$ and *longitudinal mass* $=\mu[(1-v^2/c^2)^{-1/2}]^3$. "As long as the electron moves slowly," μ is its mass (he used V instead of c, as had Lorentz). We can rewrite the right sides of each of Einstein's equations as $\mu\gamma^2$ and $\mu\gamma^3$. Lorentz had produced transverse and longitudinal masses equivalent to $\mu\gamma$ and $\mu\gamma^3$; clearly only one of these agrees with Einstein's results. Interestingly, it is the longitudinal component that matches for both theories, and it is the longitudinal component that no one would measure.

Despite the claims of dozens of textbooks,[19] biographies,[20] historical treatises,[21] and countless journal articles,[22] Eq. (1) does not appear anywhere in Einstein's June 1905 masterwork nor anywhere else in his entire published oeuvre.

The Technical Examiner Third Class had demonstrated that his theory could produce results much like those of Abraham's more traditional analysis; that may well have been his primary intention in presenting the treatment (see Sec. 10 in Ref. 1). To accomplish this unpropitious detour, Einstein assumed $F=ma$, which had the effect of bifurcating the mass. Of course, $F=ma$ holds for constant m and hardly seems an appropriate place to start a derivation of speed-dependent mass.

The June 1905 paper goes on to derive for the first time the expression for the relativistic kinetic energy. Einstein accomplished this derivation in a fortuitous way. He calculated the work W done in slowly accelerating an electron from zero to v via an electric field parallel to the x-direction (assuming no radiation). That was a lucky choice because it involved only the longitudinal mass $\mu\gamma^3$. As we will see in Sec. III A, the expression Einstein used for the force ($F_x=\mu\gamma^3 a_x$) was relativistically correct in spite of the fact that he started with $F=ma$, which was not. Integrating $dW=F_x dx=\mu\gamma^3 v dv$ immediately leads to $\mathrm{KE}=\mu c^2(\gamma-1)$, given here in only slightly modernized notation. This result was correct even though the derivation, which predates the discovery of relativistic momentum, did not begin with $\mathbf{F}=d\mathbf{p}/dt$ and so leaves something to be desired.[23]

In August 1906, five papers later, Einstein (now Technical Examiner Second Class) returned to the subject of speed-dependent mass for the last time in "On a method for the determination of the ratio of the transverse and the longitudinal mass of the electron." [24] There had been so much activity by Kaufmann and others that Einstein was drawn back to the topic, not knowing that theoretical insights were developing that would soon prompt him to abandon the concept entirely. Einstein's brief August 1906 article proposed an experiment to measure the ratio of the transverse to longitudinal masses of an electron. He calculated formulas for those ratios as predicted by the leading theories (of Abraham, Lorentz-Einstein, and a new short-lived contender by Bucherer).[25] The article ended with an entreaty to experimentalists to build and use the device, although it seems no one ever did.

Today this paper's chief significance is that it tells us that Dr. Einstein (he had received his Ph.D. in July 1905) was still invested in speed-dependent mass in mid-1906. Moreover, it reveals that even though he had not explicitly shown that the transverse mass was equal to $\mu\gamma$, he was content to have his name associated with that formulation and thereby with Lorentz, whom he deeply admired.

A. Planck, momentum, and mass

The June 1905 paper[26] contains very little dynamics, and it was to be expected that someone would further develop the subject. That person was Max Planck, then a member of the editorial board of the *Annalen der Physik*, and a very early advocate of the new theory. In less than a year Planck, who had been communicating with Einstein, was ready to publish his first memoir on relativity (in March 1906).[2] Like Einstein, he began with electrodynamics but used a different approach to force, along with the more powerful Hamilton–Lagrange formalism. Planck produced an equation for the *relativistic momentum* of a point-mass, namely, $p=mv(1-v^2/c^2)^{-1/2}$. Significantly, there was only one mass for the moving particle, and it was constant and independent of direction.

Lorentz[27] had already derived (in 1904) an expression for the electromagnetic momentum of a spherical electron (of radius R with a uniform charge distribution) whose electromagnetic mass was $m_0=e^2/6\pi Rc^2$. For such an electron, moving at a speed v, Lorentz showed that its momentum was $\mathbf{p}=\gamma m_0\mathbf{v}$. Planck was probably aware of this result, and although the physics was quite different from his far more general analysis, the agreement must have been reassuring.

Planck (March 1906)[28] went on to derive an expression for what he called the "*lebendigen Kraft*," namely, γmc^2 + constant. That's the old-fashioned term "living force," which is best translated as kinetic energy (that is the way Einstein later interpreted it).[29] When the constant is equal to $-mc^2$, this expression becomes $\mathrm{KE}=\gamma mc^2-mc^2$, which is identical to the expression Einstein had found less rigorously in 1905.[30]

Planck's insightful approach had a great influence on Einstein. After August 1906, Einstein would never again allude to transverse and longitudinal mass (even though Planck eventually did). We know roughly when this change of mind occurred. J. Stark asked Einstein to write a review article on relativity for the prestigious *Jahrbuch der Radioaktivität und Electronik* (December 1907).[31] Einstein produced a long essay in which he reconstructed his 1905 moving-electron calculation, this time in the more rigorous manner of Planck's 1906 paper.[2] As Einstein put it, "force is defined as in Planck's study," namely, as the time rate of change of momentum. Moreover, "the formulations of the equations of motion of material points, which so clearly demonstrate the analogy between these equations of motion and those of classical mechanics, are also taken from that study." By using relativistic momentum, Einstein dispensed with the transverse and longitudinal masses altogether; they simply did not enter the analysis.

From then onward (even into general relativity) Einstein focused on the centrality of the relativistic equations for energy and momentum. Beyond a single personal letter (which we shall consider), there is no record of him ever again discussing a speed-dependent mass.

Planck's next paper (September 1906)[16] was a reanalysis of Kaufmann's experiment. He came up with expressions for the electron's speed-dependent momentum as predicted alternatively by Abraham's theory[14] and by relativity. Planck's charge-to-mass ratio (ε/μ_0) was constant, and there was no mention of speed-dependent mass. Nonetheless when compared with the experimental observations, Planck's analysis still favored Abraham's theory.

The article "Zur Dynamik bewegter Systeme" (June 1907)[32] was Planck's attempt to unify electrodynamics, mechanics, and thermodynamics. It was the first paper to put forward a relativistic formulation of thermodynamics; therein Planck generalized the idea of momentum and introduced transverse and longitudinal mass. Einstein was moved by the overall effort to write Part IV, "On the mechanics and thermodynamics of systems," into his December 1907 review article.[33] Although he adopted some of Planck's methodology, Einstein never utilized the concept of speed-dependent mass and did not even mention it.

At the end of Part III of the December 1907 article, Einstein turned his attention to Kaufmann's experimental results,[34] making the following rather telling observation: "It should be mentioned that the theories of electron motion of Abraham and of Bucherer yield curves which fit the observed curves substantially better than the curves deduced from relativity theory. But it is my opinion that scant plausibility attaches to those theories, because their basic assumptions which concern the mass of the moving electron are not suggested by theoretical systems that encompass wider complexes of phenomena."[35]

The "theoretical systems that encompass wider complexes of phenomena" are no doubt the relativistic equations for energy and momentum.

The experiments of Kaufmann and others had to be reinterpreted. Any experimental deviation of the high-speed motion of electrons from that predicted classically could be understood in terms of the new dynamics, that is, in terms of the equations for relativistic momentum and energy, *mass was relativistically invariant*. The new dynamics was consistent with Lorentz's conclusions. Both predicted the same experimentally observed motion of electrons in a perpendicular **B**-field, albeit from completely different perspectives. Soon people were alluding to the Lorentz–Einstein predictions, and Einstein himself had done as much (August 1906)[25] as had Planck.

The rigorous statement of Newton's second law is $\mathbf{F}=d\mathbf{p}/dt$ not $\mathbf{F}=m\mathbf{a}$. Had Einstein begun Sec. 10 of the June 1905 treatment with $\mathbf{F}=d\mathbf{p}/dt$ and the correct relativistic form of **p**—which had not yet been discovered—he would not have arrived at two speed-dependent masses.[36]

After Planck's memoir[2] of March 1906 using $\mathbf{F}=d\mathbf{p}/dt$, anyone would have been able to show by taking the derivative of $\mathbf{p}=\gamma m\mathbf{v}$, that the relation between **F** and **a** depends on the direction of **F** with respect to **v**. When the force and velocity are perpendicular, $\mathbf{F}_\perp=\gamma m\mathbf{a}_\perp$, and when the force and velocity are parallel, $\mathbf{F}_\parallel=\gamma^3 m\mathbf{a}_\parallel$. The similarity between these and Lorentz's results no doubt convinced many that the two theories were equivalent. After 1907 there was some irony in that conclusion because it could then be said that Lorentz's theory (which involved the aether and little charged spherical electrons) had always been questionable in some regards, and that Einstein's approach (sans speed-dependent mass) was now quite correct.

In his lectures at Princeton[37] in the Spring of 1921 Einstein stated after expressing force in a form equivalent to $\mathbf{F}=d\mathbf{p}/dt=d(\gamma m\mathbf{v})/dt$ that "this equation, which was previously employed by H. A. Lorentz for the motion of electrons, has been proven to be true, with great accuracy, by experiments with β-rays."[38] He said nothing about speed-dependent mass.

Einstein derived the kinetic energy of a moving electron in terms of μ at the end of his June 1905 essay.[30] Yet after Planck's 1906 paper[39] he surely knew that early derivation was unsatisfactory. Two years later (May 1907)[40] he derived the kinetic energy of a rigid body and arrived at the same equation wherein "μ denotes its mass (in the conventional sense)." From then on, inertial mass would always be mass "in the conventional sense."

Embedded in the formula for the kinetic energy was $\mu(1-v^2/c^2)^{-1/2}$, but Einstein did not mention it specifically as a quantity of intrinsic significance, nor did he draw any conclusions about inertial mass being either speed dependent or direction-independent; those were no longer issues for him. By the end of 1907, Einstein's mechanics included a fairly sophisticated relativistic dynamics, wherein mass was invariant, although not everyone noticed. To many, the idea that mass varied with speed—as did time and length—had already become accepted and was seemingly borne out by years of experimentation.

Einstein did not publish a recantation of his little dalliance with speed-dependent mass; that was well behind him and better forgotten, nor did anyone else make any effort in the literature of the time to deal with the issue and set the record straight. There simply existed two simultaneous antithetic worldviews: to some, mass was speed dependent; to others, mass was invariant. The experimentalists pressed on, most never noticing that Einstein had quietly changed the game. It is in that unfortunate milieu that relativistic mass would soon be conceived.

B. The invention of relativistic mass

As Einstein was shunning speed-dependent mass, others were embracing it. To derive the equations for mass directly from dynamics, Gilbert Lewis and Richard Tolman[41] (1908–1912) considered various kinds of collisions. These treatments contain the unjustified assumption that momentum has the classical form $\mathbf{p}=m\mathbf{v}$ (something Einstein no longer accepted). Inexplicably, Planck's reformulation, $p=\gamma mv$, was completely ignored.

Undaunted, Lewis and Tolman coupled that conjecture with the two classical laws of conservation of mass and momentum, applied in a manner that was consistent with the coordinate transformations of Lorentz. They derived a single directionally independent expression for the mythical speed-dependent mass, $m=m_0(1-v^2/c^2)^{-1/2}$. Among the many believers this derivation was a great success. It did not matter that it was a blend of classical and relativistic ideas; the end result was just what some people were looking for.

Paraphrasing Tolman (1917),[42] when force is defined as the time rate of change of momentum, and momentum is Newton's momentum, not Planck's, then a single speed-dependent mass independent of direction, $m_r(v)$, results. Because this expression was identical to the Lorentz–Einstein mass $m_0\gamma$, the concept that would become known as relativistic mass soon muted any mention of transverse and longitudinal mass.

In time, the improved data of Bucherer (1909),[43] Neumann (1914),[44] and others[45] seemed to some to establish the veracity of $m_r=m_0(1-v^2/c^2)^{-1/2}$, but to those who followed Einstein, that was an illusion. We do not know who was first to call it relativistic mass, it may well have been Born (1920),[46] but it was not long before $m_r(v)$ was codified as a central part of Einstein's dynamics in spite of his unwavering, albeit silent, disapproval.

In his many papers (including the unpublished manuscript in 1912), lectures (for which transcripts survive)[37] and books, Einstein produced relativistic expressions for the total energy, kinetic energy, rest energy, and momentum, but he never explicitly derived $m_r=m_0(1-v^2/c^2)^{-1/2}$. Although the juxtaposition of m and $(1-v^2/c^2)^{-1/2}$ showed up frequently in his work, Einstein did not combine the two terms into the single concept of relativistic mass. Whenever he used the word "mass" after 1906, it was, as he put it, in the conventional sense. For example, in his Gibbs Lecture in Pittsburgh (1934), he referred to "m (as) the rest mass or, simply, the mass."[47]

In 1948 Albert Einstein wrote in a private letter: "It is not proper to speak of the mass $M=m(1-v^2/c^2)^{-1/2}$ of a moving body, because no clear definition can be given for M. It is better to restrict oneself to the rest mass, m. Moreover one may certainly use the expressions for momentum and energy when referring to the inertial behavior of rapidly moving bodies."[48] And that is exactly what he did after 1906 until his death in 1955.[49] Unfortunately, by the mid-20th century, the conceptual edifice built on the notion of speed-dependent mass and crowned by $E=mc^2$ had become the well established consensus in spite of the great man's wishes.

C. The origins of $E_0=mc^2$

Three months after the June 1905 essay,[50] Einstein published a short but important paper concerning energy. Over the years he went on to develop the nomenclature of relativistic energy as well as the notation, but both were still rudimentary in 1905. When Einstein wrote the paper, "Does the inertia of a body depend upon its energy content?" (September 1905),[51] he did not yet make the distinction between "total energy" E and "rest energy" E_0, and neither can be found in that essay although "energy content" (*energieinhalt*) seems to be his equivalent of "internal energy." Moreover he did not distinguish explicitly between rest mass and relativistic mass; the latter term had not yet been conceived. Accordingly there is great opportunity here for misinterpretation, which has occurred frequently since that paper was published. Still, this article is most often cited as the origin of $E=mc^2$ and that makes an accurate exegesis all the more important. Fadner (1988)[52] correctly asserted that "while Einstein was the first to write $E_0=m_0c^2$ (in our notation), he did not actually write $E=mc^2$ in these early papers." And he did not do so, not as an oversight, but because he knew better. Okun (1989) has astutely pointed out[8] that one of the issues of contention in the physics community today is whether the correct energy equation is $E=mc^2$ wherein m is relativistic mass or $E_0=mc^2$ wherein m is (rest) mass. We will show that Einstein always embraced the latter formulation.

In the September 1905 memoir,[53] Einstein imagined a body at rest emitting two plane waves of light in opposite directions where "all the while, the body shall stay at rest." Using results from his June paper,[1] he showed that "the mass of a body is a measure of its energy content." In other words, "if the energy changes by L, the mass changes in the same sense by" L/c^2. Some contemporary writers have reinterpreted this phrase as $\Delta E/c^2$ and concluded therefore that $\Delta E=\Delta mc^2$, which naturally leads to $E=mc^2$, where E is total energy and m is relativistic mass, a concept not even in existence in 1905. Einstein certainly did not put forward that line of reasoning.

The "body" in Einstein's analysis was at rest before and after it emitted. That is why he had it send out two waves in opposite directions. Thus designating the object's rest mass as m_0, it follows that $\Delta m_0=L/c^2$. Because $E=E_0$, it must be that $L=\Delta E_0$ and $\Delta E_0=\Delta m_0c^2$. Once it is established that there is no other mass than the rest mass, we should write this relation as $\Delta E_0=\Delta mc^2$, which leads to the very different conclusion, $E_0=mc^2$. This is what Einstein meant here, and it is what he meant on those few occasions[54] when he wrote $E=mc^2$. He consistently made it clear that the body he was talking about was at rest so that E was the energy of the system at rest (what would come to be written as E_0).

The physical significance of the concept of rest energy had not yet been enunciated by Einstein or anyone else. Thus it was reasonable for him to talk about the energy of an object at rest and write it as E rather than E_0. Einstein only got down to examining the notion that we call rest energy[55] in 1907 and did not consistently designate it as E_0 until much later.

In May of 1906 Einstein published a short paper entitled "The principle of conservation of motion of the center of gravity and the inertia of energy." [56] It explored the relation between energy and mass, mindful of some earlier work (1900) by Poincaré[57] in which he had produced an equation suggestive of $E=mc^2$. It is important to look at this 1906 paper carefully because it too is often erroneously cited as the place where Einstein introduced $E=mc^2$.

Einstein's 1906 study[58] was problematic in that he utilized massless objects at a time when no one knew anything about their dynamics. His Gedanken experiment involved a "stationary hollow cylinder freely floating in space" whose end caps (left and right) were A and B. A pulse of light of energy S—we use E here instead—leaves A and travels to B. From Maxwell's theory, upon emission of the pulse, the A-cap experiences a backward force and the cylinder recoils to the left. The pulse is soon absorbed at B and the displaced cylinder comes to rest.

Einstein then transferred E to a mythical massless "carrier body" at B. Because he uses a body and not a point, the energy can be taken up internally by the carrier. The carrier first absorbs E and is then moved back to A; the idea is to return the cylinder to its original state. Assume the carrier now has mass and momentum because it contains E. This is the rest energy of the carrier, although Einstein did not call it that. The cylinder recoils to the right on the departure of the carrier body and comes to a stop at its original location when that carrier stops at A. There the energy is off-loaded to the A-cap from whence it came. To return the system to its original configuration, Einstein sent the massless carrier, now sans E, back to B. Because it is without E and also without mass, it does not carry momentum and does not shift the cylinder. Everything is returned to its original state, and the system has gone through a complete cycle and is unchanged provided mass is associated with the rest energy of the otherwise massless carrier body.

If that were not the case and the carrier "remains massless even after it has absorbed the amount of energy" E, the cylinder would not shift to the right when the carrier leaves B. Hence, the system could be cycled back to its original configuration except the cylinder would have advanced to the left in the process. That advance could be repeated endlessly, moving the cylinder along without the action of an external force, thereby violating the basic principles of mechanics. The conclusion is clear: "any energy E (transferred into the carrier body at rest) possesses the inertia E/V^2;" that is, $m=E/c^2$, where E is a specific amount of internal energy. Because it is internal or rest energy, the equation is properly written as $m=E_0/c^2$.

Our concern here is less with the physics of that thought experiment than with Einstein's views on mass and energy. We conclude that as of mid-1906 he embraced $E_0=mc^2$ and he did not intend to establish, derive, or sanction $E=mc^2$.

This thought experiment has often been redesigned in recent years[59] and employed to claim that Einstein proved that the total energy E equals mc^2 where m is relativistic mass. Not so! In fact, here E is the quantity of energy originally in the light pulse, the quantity added at B to the carrier body at rest, and thus it is rest energy and corresponds to rest mass (m) via $E_0=mc^2$.

The discussion of energy took on a decidedly more modern tone in Einstein's May 1907 paper, "On the inertia of energy required by the relativity principle." [60] This paper began, "the principle of relativity, in combination with Maxwell's equations, leads to the conclusion that the inertia (that is, inertial mass) of a body increases or decreases with its energy content in a completely determined way." Einstein again used the phrase energy content, something very close to rest energy, a term still not yet articulated.

In Sec. IV of Ref. 60 Einstein treated a collection of force-free, differently moving point masses. He next considered a frame wherein the net momentum was zero (which can always be found for objects having mass). There the kinetic energy of the system moving as a whole vanished. In that frame he designated the energy as E_0 (actually ε_0), and although he did not refer to it that way, this quantity is the rest energy. He then set E_0 equal to μV^2, still using V for c, which in modern notation translates into $E_0=mc^2$, wherein m is rest mass.

As seen from a coordinate frame with respect to which the multiparticle system was moving uniformly, Einstein maintained[61] that it had a total energy $E=\gamma mc^2$. He was very clear here; the total energy of the system was $E=\gamma mc^2$, and $E=mc^2$ does not appear anywhere in the discussion.

The paper concludes, "Thus, a system of moving mass points—taken as a whole—has more inertia the faster the mass points move relative to each other." It is the internal "relative" motion, the internal kinetic energy, and not the uniform motion of the system as a whole or the external kinetic energy that contributes to the inertial mass of the system. If external kinetic energy contributed, mass would be speed dependent, and Einstein had rejected that idea.

D. The meaning of rest energy

Einstein did not develop the modern concept of rest energy until his long review article[62] of December 1907. There he considered a compound physical system and showed that it "behaves like a material point of mass M, where M depends on the system's energy content E_0 according to the formula $M=\mu+E_0/c^2$." The quantity M is the mass we measure (that is, the "apparent" mass), μ is the "actual" mass (that is, the rest mass of its various parts), and E_0 is the internal energy (kinetic plus potential energy) of the system. This paper is still an early one and the nomenclature was evolving such that E_0 is not yet the more inclusive rest energy familiar to us. Einstein explains, "Since we can arbitrarily assign the zero-point of E_0, we are not even able to distinguish between a system's 'actual' mass and its 'apparent' mass without arbitrariness. It seems far more natural to consider any inertial mass as a reserve of energy." [63]

Inasmuch as the zero of E_0 is arbitrary, the energy μc^2 can be folded into E_0, whereupon $M=E_0/c^2$, and with that gesture E_0 becomes our modern-day rest energy. This extension of the concept of internal energy was a major advance in that henceforth the rest energy of a system of interacting particles would explicitly embrace the internal potential energy, internal kinetic energy, and the mass-energy of the once free particles now constituting the system. Further, because "any inertial mass" is "a reserve of energy," we can expect that rest energy would be able to be converted into some other form of energy, viz., kinetic energy. That ability to transform is a central aspect of energy.

In 1920 when Einstein wrote, "we can look at mc^2 as the energy of the mass point when it is at rest,"[64] the nomenclature was still somewhat incomplete. In his May 1921 lectures at Princeton, he said that $E_0 = mc^2$ and with c set equal to one, "the energy, E_0, of a body at rest equals its mass."[65] By 1934 Einstein was using the terms rest energy and total energy as is done nowadays, but by then much of the scientific world had already accepted $E = mc^2$ as if it were part of the catechism of relativity.

E. Conservation and conversion

Today's physics literature contains dozens of articles maintaining that relativistic mass is conserved and that as a result there is no such thing as the conversion of mass into energy and vice versa.[66] This issue, which comes down to the fundamental equivalence of mass and energy, is an ongoing concern of tremendous philosophical and pedagogical importance. Moreover, Einstein's position on these matters has often been misrepresented. Although he never spoke publicly about his views on relativistic mass (perhaps because in 1905 he had contributed to the confusion), he was much more forthcoming on the issues of conservation of mass and the interconvertibility of mass and energy.

Einstein was unequivocally against the traditional idea of conservation of mass. He had concluded that mass and energy were essentially one and the same; "inert mass is simply latent energy."[67] He made his position known publicly time and again:[68] "the inertial mass of a body depends on its energy content…. This theorem overturns the principle of the conservation of mass, or, rather, fuses it with the principle of conservation of energy into a single principle."[69]

Einstein was not the first person to suggest that mass might be transformed into energy. Frederick Soddy in 1904, impressed by the amount of energy available from radioactivity, presciently concluded that its source was the mass of the sample itself.[70] According to Pais, "Einstein had in mind the loss of weight resulting from radioactive transformations," whenever he talked about "the mass-energy equivalence."[71] As early as 1907 Einstein put it in terms that cannot be misinterpreted: "it is possible that radioactive processes will be detected in which a significantly higher percentage of the mass of the original atom will be converted into the energy of a variety of radiations than in the case of radium."[72] We can go to the AIP history website[73] and hear Einstein say (1948) in a gentle unassuming voice: "It followed from the special theory of relativity that mass and energy are both but different manifestations of the same thing—a somewhat unfamiliar conception for the average mind. Furthermore, the equation E is equal (to) mc^2, in which energy is put equal to mass, multiplied by the square of the velocity of light, showed that very small amounts of mass may be converted into a very large amount of energy and vice versa."

The operative word here is "converted."

IV. CONCLUSIONS

After Planck introduced relativistic momentum in 1906, Einstein abandoned speed-dependent mass. From then on mass was invariant. When the term relativistic mass [Eq. (1)] became popular, Einstein silently eschewed it. By then he was well into the general theory and for him m was "an invariant (tensor of rank zero)."[74]

Einstein often reminisced about the contributions of special relativity. Not surprisingly he never mentioned relativistic mass, but instead opined that the "most important upshot" was the discovery "that inert mass is simply latent energy."[75] On several occasions he wrote the expression $E_0 = mc^2$, wherein m was always invariant mass—the only mass—no need any longer to call it rest mass or subscript it with a zero.

Einstein never derived $E = mc^2$ without making it clear that "E is the energy contained in a stationary body."[76] Still, those who accepted relativistic mass had given the public a mantra it could recite, namely, $E = mc^2$, which became a pop-culture icon, even if its meaning was muddled. That Einstein was sensitive to the misleading nature of that equation—as it had come to be widely understood—is evident in his comment (1946): "It is customary to express the equivalence of mass and energy (though somewhat inexactly) by the formula $E = mc^2$…."[76]

[a]Electronic mail: genehecht@aol.com

[1]*The Collected Papers of Albert Einstein, The Swiss Years: Writings, 1900–1909*, Vol. 2, translated by Anna Beck (Princeton U. P., Princeton, NJ, 1989), pp. 140–171. This version is true to the original in notation Also in English is A. Einstein, *The Principle of Relativity* (Dover, New York, 1923), pp. 35–65, but here the notation has been modernized.

[2]Max Planck, "Das Prinzip der Relativität und die Grundgleichungen der Mechanik," Verh. Dtsch. Phys. Ges. **8**, 136–141 (1906).

[3]Lev B. Okun, "The concept of mass (mass, energy, relativity)," Sov. Phys. Usp. **32**, 629–638 (1989) addressed several important issues. W. Rindler, "Putting to rest mass misconceptions," Phys. Today **43**(5), 13 (1990); M. Vandyck, "Putting to rest mass misconceptions," *ibid.* **43**(5), 13 (1990); K. B. Kolb, "Mass and energy," Am. J. Phys. **34**(8), 705 (1966); C. G. Adler, "Does mass really depend on velocity, dad?" *ibid.* **55**(8), 739–743 (1987); G. J. Aubrecht II, "Comment on 'Apparatus to measure relativistic mass increase'," *ibid.* **72**(7), 970–971 (2004); D. Easton, "Mass is not destroyed," Phys. Teach. **26**(4), 230 (1988); A. Hobson, "Teaching $E = mc^2$: Mass without mass," *ibid.* **43**(2), 80–82 (2005); S. Carson, "Relativistic mass," Phys. Educ. **33**(11), 343–344 (1998); J. W. Warren, "The mystery of mass-energy," *ibid.* **11**(1), 52–54 (1976); K. Atkin, "Clarifying the concept," *ibid.* **35**(9), 319 (2000).

[4]Albert Einstein, "$E = Mc^2$," *Ideas and Opinions* (Dell, New York, 1976), p. 330.

[5]J. J. Thomson, "On the electric and magnetic effects produced by the motion of electrified bodies," Philos. Mag. **11**, 229–249 (1881).

[6]For general reference, see Max Jammer, *Concepts of Mass in Classical and Modern Physics* (Dover, New York, 1997).

[7]G. F. C. Searle, "On the steady motion of an electrified ellipsoid," Philos. Mag. **44**, 329–341 (1897). See also O. Heaviside, "On the electromagnetic effects due to the motion of electrification through a dielectric," *ibid.* **27**, 324–339 (1889).

[8]L. B. Okun, "The concept of mass," Phys. Today **42**(6), 31–36 (1989) points out, "I have not found in the papers I have read (Ref. 7) any suggestion that mass depends on velocity."

[9]To see how electromagnetic mass follows from Searle's expression for field energy, read the 1901 paper by Walter Kaufmann, "Magnetic and electric deflectability of the Becquerel rays and the apparent mass of the electron," Gött. Nachr. **2**, 143–513 (1901), translated in Henry Boorse and Lloyd Motz, *The World of the Atom* (Basic Books, New York, 1966), pp. 506–512.

[10]H. A. Lorentz, "Simplified theory of electric and optical phenomena in moving systems," Konikl. Akad. Wetenschap. Proc. **1**, 427–442 (1899). See Stanley Goldberg, "The Lorentz theory of electrons and Einstein's theory of relativity," Am. J. Phys. **37**(10), 982–994 (1969) as well as Kenneth F. Schaffner, "The Lorentz electron theory of [sic] relativity," *ibid.* **37**(5), 498–513 (1969).

[11]James Cushing, "Electromagnetic mass, relativity and the Kaufmann experiments," Am. J. Phys. **49**(12), 1133–1149 (1981) (that excellent paper also contains a highly readable exegesis of Abraham's theory and its relation to Kaufmann's experiment).

[12]Arthur Miller, *Albert Einstein's Special Theory of Relativity* (Addison-Wesley, Reading, MA, 1981), pp. 47–55.

[13] Reference 1, p. 140.

[14] Max Abraham, "Prinzipien der Dynamik des Elektrons," Ann. Phys. **10**, 105–179 (1903).

[15] H. A. Lorentz, "Electromagnetic phenomena in a system moving with any velocity less than that of light." A translation of this paper appears in A. Einstein, *The Principle of Relativity* (Dover, New York, 1923), pp. 11–34. Alternatively see, Konikl. Akad. Wetenschap. Proc. **6**, 809–831 (1904).

[16] Max Planck, "Die Kaufmannschen Messungen der Ablenkbarkeit der β-Strahlen in ihrer Bedeutung für die Dynamik der Elektronen," Phys. Z. **7**, 753–761 (1906). Also see Ref. 1, Anna Beck, "On the relativity principle and the conclusions drawn from it," p. 283.

[17] John Stachel, *Einstein from 'B' to 'Z'* (Birkhäuser, Boston, MA, 2002); Don Howard and John Stachel, *Einstein The Formative Years, 1879–1909 Z'* (Birkhäuser, Boston, MA, 1998).

[18] John Earman, Clark Glymour, and Robert Rynasiewicz, "On writing the history of special relativity," Phil. Soc. Am. **2**, 403–416 (1982). Robert Rynasiewicz, "The optics and electrodynamics of 'On the electrodynamics of moving bodies'," Ann. Phys. (Leipzig) Supplement **14**, 38–57 (2005); Gerald Holton, "On the origins of the special theory of relativity," Am. J. Phys. **28**, 627–636 (1960).

[19] L. H. Greenberg, *Physics with Modern Applications* (Saunders, Philadelphia, 1978), p. 456; Marshall Burns, *Modern Physics for Science and Engineering* (Harcourt Brace Jovanovich, New York, 1988), p. 103 calls Eq. (1) "Einstein's mass equation." H. Semat, *Introduction to Atomic and Nuclear Physics* (Rinehart, New York, 1958), p. 46; F. W. Sears, M. W. Zimansky, and H. D. Young, *University Physics*, 5th ed. (Addison-Wesley, Reading, MA, 1976), p. 131; R. A. Serway and J. S. Faugh, *College Physics*, 3rd ed. (Saunders, New York, 1992), p. 894.

[20] There are hundreds of popular books that get it wrong, the latest of which are Walter Isaacson, *Einstein* (Simon and Schuster, New York, 2007), p. 130 and Hans Ohanian, *Einstein's Mistakes: The Human Failings of Genius* (Norton, New York, 2008).

[21] Max Jammer, *Concepts of Mass in Contemporary Physics and Philosophy* (Princeton U. P., Princeton, NJ, 2000), p. 42, states that Einstein's first paper "introduced the notion of relativistic mass," which Jammer had earlier carefully defined as $m_r = m_0(1-v^2/c^2)^{-1/2}$. Also see Stanley Goldberg, *Understanding Relativity: Origin and Impact of a Scientific Revolution* (Birkhäuser, Boston, 1984).

[22] Ling Tsai, "The relation between gravitational mass, inertial mass, and velocity," Am. J. Phys. **54**(4), 340–342 (1986); Sivio Bergia, "Einstein and the birth of special relativity," in *Einstein: A Centenary Volume*, edited by A. P. French (Harvard U. P., Cambridge, MA, 1979).

[23] Edmund Whittaker, *A History of the Theories of Aether and Electricity*, Vol. 2 (Dover, New York, 1989), p. 44, asserted "the study of relativistic dynamics was begun in 1906, when Max Planck found the equations… of motion." Since Einstein's 1905 derivation was weak, Whittaker makes a valid point.

[24] Reference 1, pp. 207–210, "On a method for the determination of the ratio of the transverse and the longitudinal mass of the electron."

[25] Reference 1, p. 210.

[26] Reference 1, p. 167.

[27] Reference 15, Sec. 9. Also see Olivier Darrigol, *Electrodynamics from Ampere to Einstein* (Oxford U. P., Oxford, 2000), pp. 360–390.

[28] Reference 2, p. 119.

[29] Reference 1, "On the relativity principle and the conclusions drawn from it," pp. 278–280.

[30] Reference 1, p. 170.

[31] Reference 1, p. 252–311.

[32] Max Planck, "Zur Dynamik bewegter Systeme," Sitzungsber. Preuss. Akad. Wiss. **1907**, 542–570. Also see Max Planck, *Eight Lectures on Theoretical Physics* (Dover, Mineola, NY, 1998), pp. 97–130.

[33] Reference 1, p. 284.

[34] The original German version can be found in *The Collected Papers of Albert Einstein, The Swiss Years: Writings, 1900–1909*, Vol. 2, edited by John Stachel (Princeton U. P., Princeton, NJ, 1989), pp. 432–484.

[35] Reference 1, p. 284. Although this translation is the "official" one, in this instance it is terribly misleading because it translates "*die Masse*" as "the dimensions." For the German text, see Ref. 34, p. 461; H. M. Schwartz, "Einstein's comprehensive 1907 essay on relativity, part II," Am. J. Phys. **45**(9), 811–817 (1977) translates the passage accurately.

[36] Reference 34, p. 309, footnote 41. The 1913 reprint of Einstein's June 1905 paper contained an appended note: "The definition of force given here is not advantageous as was first noted by M. Planck. It is instead appropriate to define force in such a way that the laws of momentum and conservation of energy take the simplest form."

[37] *The Collected Papers of Albert Einstein, The Berlin Years: Writings, 1918–1921*, Vol. 7, translated by Alfred Engel (Princeton U. P., Princeton, NJ, 2002); A. Einstein, "Four lectures on the theory of relativity, held at Princeton University in May 1921" (1922), pp. 261–368.

[38] Reference 37, p. 307.

[39] Reference 2.

[40] Reference 1, p. 240.

[41] G. N. Lewis, "A revision of the fundamental laws of matter and energy," Philos. Mag. **16**(11), 705–717 (1908); G. N. Lewis and R. C. Tolman, "The principle of relativity and non-Newtonian mechanics," *ibid.* **18**, 510–523 (1909); R. C. Tolman, "Note on the derivation from the principle of relativity of the fifth equation of the Maxwell–Lorentz theory," *ibid.* **21**, 296–301 (1911); "Non-Newtonian mechanics: The mass of a moving body," Philos. Mag. **23**, 375–380 (1912). For a different approach, see P. C. Peters, "An alternative derivation of relativistic momentum," Am. J. Phys. **54**(9), 804–808 (1986).

[42] Richard C. Tolman, *The Theory of the Relativity of Motion* (U. of California Press, Berkeley, 1917), pp. 62–67.

[43] A. H. Bucherer, "Die experimentelle Bestätigung des Relativitätsprinzips," Ann. Phys. **28**, 513–536 (1909).

[44] G. Neumann, "Die träge Masse schnell bewegter Electronen," Ann. Phys. **45**, 529–579 (1914).

[45] See W. Pauli, *Theory of Relativity* (Pergamon, New York, 1958), p. 83, first published in 1921. Also see Ref. 6, pp. 166–168.

[46] Max Born, *Einstein's Theory of Relativity* (Dover, New York, 1962), first published in 1920.

[47] A. Einstein, "Elementary derivation of the equivalence of mass and energy," Bull. Am. Math. Soc. **41**, 223–230 (1935); David Topper and Dwight Vincent, "Einstein's 1934 two-blackboard derivation of energy-mass equivalence," Am. J. Phys. **75**(11), 978–983 (2007). Topper and Vincent point out that "in this lecture Einstein stayed well clear of defining the relativistic mass parameter, $m_R, \ldots$" That should not have surprised anyone. He always stayed clear of it. Their article contains a photo of Einstein at the 1934 Pittsburgh lecture standing next to a blackboard on which is written $E_0 = m$ and he had set $c = 1$.

[48] During April, May, and June of 1948, *Harper's* magazine published a series of articles by Lincoln Barnett, a well-known science writer. Therein Barnett talked about "Einstein's equation [Eq. (1)] giving the increase of mass with velocity…". On June 19, 1948, Einstein sent Barnett a letter (written in German) to set the record straight on his position against relativistic mass. See Ref. 8, p. 32 for a photograph of the letter, which is now in the possession of the Hebrew University of Jerusalem, Israel.

[49] A. Einstein and L. Infeld, *The Evolution of Physics* (Simon and Schuster, New York, 1938), p. 205, distinguish between inertia (the measure of a body's resistance to a change in motion) and mass without explicitly saying as much. As Einstein and Infield put it, "If two bodies have the same rest mass, the one with the greater kinetic energy resists the action of an external force more strongly." It would have to if c is to be its limiting speed.

[50] Reference 1, pp. 140.

[51] Reference 1, p. 172.

[52] W. L. Fadner, "Did Einstein really discover '$E = mc^2$'?" Am. J. Phys. **56**, 114–122 (1988). Also see John Stachel and Roberto Torretti, "Einstein's first derivation of the mass-energy difference," *ibid.* **50**(8), 760–763 (1982).

[53] Reference 1, pp. 172–174.

[54] Albert Einstein, *Out of My Later Years* (Citadel, Secaucus, NJ, 1956), p. 116.

[55] Reference 1, pp. 286–287.

[56] Reference 1, pp. 200–206.

[57] H. Poincaré, Lorentz-Festschrift, "La théorie de Lorentz et le principe de reaction," Arch. Neerl. Sci. Exactes Nat., Ser. 3A **2**, 252–278 (1900). Also see Stanley Goldberg, "Henri Poincare and Einstein's theory of relativity," Am. J. Phys. **35**(10), 934–944 (1967) and Edmund Whittaker in Ref. 23, Vol. 2, p. 51.

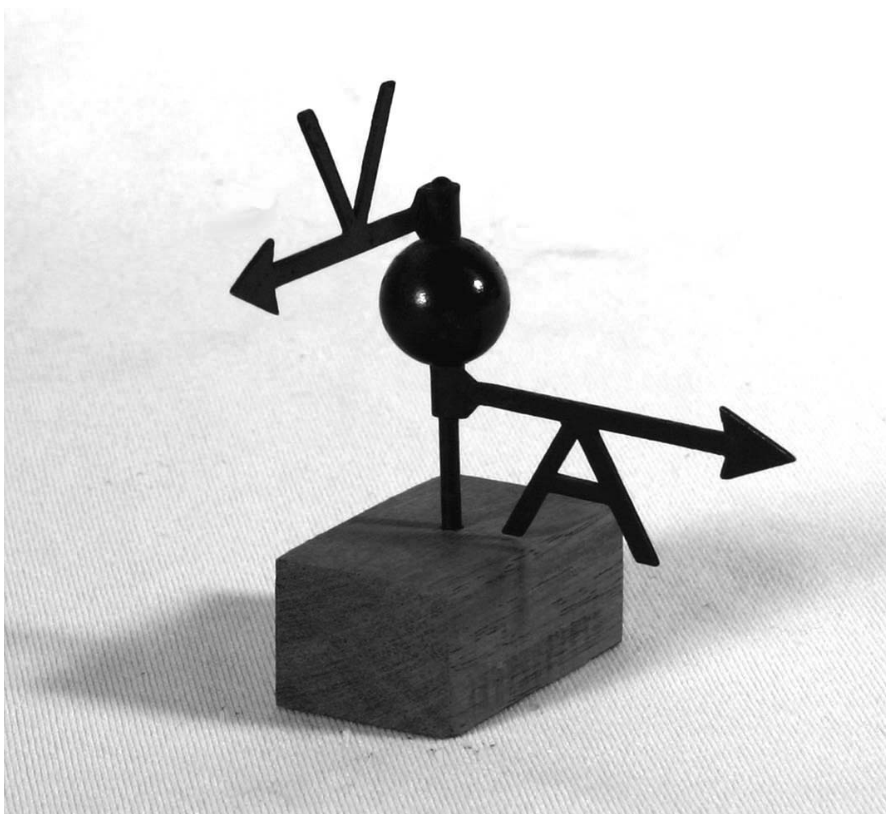
[58] Reference 1, p. 200.

[59] For example, see A. P. French, *Special Relativity* (Norton, New York, 1968), p. 17, and Adel Antippa, "Inertia of energy and the liberated photon," Am. J. Phys. **44**(9), 841–844 (1976).

[60] Reference 1, pp. 238–250.

[61] Eugene Hecht, "Energy and change," Phys. Teach. **45**(2), 88–92 (2007).

[62]Reference 1, pp. 252–311.
[63]Reference 1, p. 287.
[64]Reference 37, "Fundamental ideas and methods of the theory of relativity, presented in their development," p. 129.
[65]Reference 37, p. 306.
[66]C. R. Eddy, "A relativistic misconception," Science **104**, 303–304 (1946); A. J. O'Leary, "Comments on 'A relativistic misconception'," *ibid.* **104**, 401 (1946). Also Hermann Bondi and C. B. Spurgin, "Energy and mass," Phys. Bull. **38**, 62–63 (1987). See the counterargument by Rudolf Peierls, "Mass and energy," *ibid.* **38**, 127–128 (1987). Reference 3; D. Easton, A. Hobson, S. Carson, J. W. Warren, K. Atkin, and Ralph Baierlein, "Teaching $E=mc^2$," Am. J. Phys. **57**(5), 391–392 (1989); R. Bauman, "Mass and energy: The low-energy limit," Phys. Teach. **32**(9), 340–342 (1994); R. Baierlein, *Newton to Einstein: The Trail of Light* (Cambridge U. P., Cambridge, 1992), pp. 236–247; Peter Stansbury, "The equivalence of mass and energy," Phys. Educ. **35**(11), 378–379 (2000) maintained that "it is impossible to change mass into energy, just as it is impossible to change bricks into apples." A. Hobson, Ref. 3, more recently (2005) suggested that "mass is never converted into energy, and energy is never converted into mass, because energy (and hence mass) can never be created or destroyed."
[67]Reference 54, p. 57.
[68]This is the way Einstein put it in Science, May 24, 1940: "There followed also the principle of the equivalence of mass and energy, with the laws of conservation of mass and energy becoming one and the same. This quote appears in the essay "The fundamentals of theoretical physics," Ref. 4, p. 321. See Science, New Series, **91** No. (2369), 487–492 (1940).
[69]"The theory of relativity," *The Collected Papers of Albert Einstein, Vol. 3, The Swiss Years: Writings, 1909–1911*, translated by Anna Beck (Princeton U. P., Princeton, NJ, 1989), pp. 349–350.
[70]Frederick Soddy, *Radio-Activity: An Elementary Treatise, from the Standpoint of the Disintegration Theory* (Van Nostrand, New York, 1904), p. 164, prophetically wrote, "on the nature of inertia and its possible dependence on the internal energy of the atom, it is indicated that the law of conservation of mass will not apply to the case of atomic transformation." And on p. 180, he asserted, "atomic mass must be regarded as a function of the internal energy, and the dissipation of the latter in radio-activity occurs at the expense, to some extent at least, of the mass of the system.
[71]Abraham Pais, *Subtle is the Lord* (Oxford U. P., Oxford, 1982), p. 149.
[72]Reference 1, p. 288.
[73]The AIP history website is ⟨www.aip.org/history/einstein/voice1.htm⟩.
[74]Reference 37, p. 277.
[75]Reference 54, p. 57.
[76]Reference 4, p. 330.



Centripetal Vectors. This pair of velocity and acceleration vectors is part of the Simple Harmonic Motion Demonstration Apparatus listed in the 1929 Central Scientific Company for $12.00. The entire apparatus consisted of a metal disk about 16 cm diameter, with the vector pair mounted 7 cm from the center. The "V" was oriented in the direction of the tangential velocity, and the "A" pointed inward, showing the direction of the centripetal acceleration. The system was illuminated from the side by a broad beam of light, and the shadows of the letters were cast on the wall as the ball moved back and forth in simple harmonic motion. The device is in the Greenslade Collection. (Photograph and Notes by Thomas B. Greenslade, Jr., Kenyon College)